

TIM OLIMPIADE MATEMATIKA INDONESIA

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Selasa, 3 Mei 2011

IMO SHORTLIST 2005

Algebra

1. **A1(ROM)** Find all monic polynomials $p(x)$ with integer coefficients of degree two for which there exists a polynomial $q(x)$ with integer coefficients such that $p(x)q(x)$ is a polynomial having all coefficients ± 1

2. **A2(BUL)** Let \mathbb{R}^+ denote the set of positive real numbers. Determine all functions $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that

$$f(x)f(y) = 2f(x + yf(x))$$

for all positive real numbers x and y

3. **A3(CZE)** Four real numbers p, q, r, s satisfy

$$p + q + r + s = 9 \text{ and } p^2 + q^2 + r^2 + s^2 = 21$$

Prove that $ab - cd \geq 2$ holds for some permutation (a, b, c, d) of (p, q, r, s)

4. **A4(IND)** Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfying the equation

$$f(x + y) + f(x)f(y) = f(xy) + 2xy + 1$$

for all real x and y

5. **A5(KOR)** Let $x, y,$ and z be positive real numbers such that $xyz \geq 1$. Prove that

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} + \frac{y^5 - y^2}{y^5 + x^2 + z^2} + \frac{z^5 - z^2}{z^5 + x^2 + y^2} \geq 0$$