TIM OLIMPIADE MATEMATIKA INDONESIA

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1. Prove that if $x, y, z > 0$ satisfy $xy + yz + zx + 2xyz = 1$, then

$$xyz \leq \frac{1}{8} \text{ and } xy + yz + zx \geq \frac{3}{4}$$

2. (a) Let $x, y, z > 0$ and $xyz = x + y + z + 2$, then

$$xy + yz = zx \geq 2(x + y + z) \text{ and } \sqrt{x} + \sqrt{y} + \sqrt{z} \leq \frac{3}{2}\sqrt{xyz}$$

   (b) If $x, y, z > 0$ and $xy + yz + zx = 2(x + y + z)$. Prove that

$$xyz \leq x + y + z + 2$$

3. The excircles of triangle ABC touch the sides AB, BC, CA at points M, N, P respectively. Let I, O be the incenter and circumcenter of triangle ABC. Prove that if $AMNP$ is a cyclic quadrilateral then

   (a) the points M, P, I are collinear

   (b) the points I, O, N are collinear

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4. Do there exist unbounded sequences $a_1, a_2, \cdots$ of positive real numbers such that for any $n \geq 1$ satisfy

$$a_{n+2} = \frac{1}{2009}\left(a_n + \frac{1}{a_{n+1}}\right)$$

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5. In a country some of the town are connected with roads. Denote by t the least positive integer for which there exist a town from which any town can be reached by at most t roads. Prove that there exist towns $A_1, A_2 \cdots, A_{2t-1}$, such that for any $i \neq j$, $i = 1, 2, \cdots, 2t-2$, $j = 2, 3, \cdots, 2t-1$ towns A_i and A_j are connected by a road if and only if $i + 1 = j$

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