TIM OLIMPIADE MATEMATIKA INDONESIA

PEMBINAAN TAHAP 3 CALON PESERTA IMO 52

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1. Let be given $a, b, c > 0$ satisfying $a + b + c = 1$. Prove that

$$a^4 b + b^4 c + c^4 a \leq \frac{256}{3125}$$

2. Find all pairs of positive integers m and n for which

$$\sqrt{m^2 - 4} < 2\sqrt{n} - m < \sqrt{m^2 - 2}$$

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3. Let ABC be an acute angled triangle such that $\angle BAC = \frac{\pi}{4}$. Let CD be an altitude and P be arbitrary interior point of segment CD. Show that $AP \perp BC$ if and only if $AP = BC$

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4. Find the least positive number x with the following property : if a, b, c, d are arbitrary positive numbers such that whose product is 1, then

$$a^x + b^x + c^x + d^x \geq \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$$

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5. Two circles k_a and k_b are given, whose center lie on the legs of lengths a and n, respectively, of a right triangle. Both circles are tangent to the hypotenuse. Denote the radii of the circle by r_a and r_b. Find the greatest real number p such that the inequality

$$\frac{1}{r_a} + \frac{1}{r_b} \geq p \left(\frac{1}{a} + \frac{1}{b} \right)$$

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6. The natural number n is called *cute* if there exist exactly four natural numbers k such that $n + k$ divides $n + k^2$

   (a) Show that 58 is *cute* and find the corresponding numbers k

   (b) Show that an even number $n = 2p$ where $p \geq 3$ is *cute* if and only if both p and $2p + 1$ are primes.

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7. In each vertex of a regular n-gon $A_1 A_2 \cdots A_n$ there lies a certain number of coins : in the vertex A_k there are exactly k coins for all $1 \leq k \leq n$. We choose two coins and move each of them into one neighboring vertices in such a way that one is moved clockwise and the other one anti-clockwise. Decide for which n it is possible to achieve, after a finite number of steps, that for each k, there are exactly $n + 1 - k$ coins in the vertex A_k

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