TIM OLIMPIADE MATEMATIKA INDONESIA

PEMBINAAN TAHAP 3 CALON PESERTA IMO 52

Minggu, 8 Mei 2011

1. Find all integer k such that for every positive integer n the number $kn+1$ and $4n+1$ are relatively prime

   Hungary

2. Let n be positive integer. Prove that if the sum of all positive divisors of n is a power of 2, then the number of these divisors is also a power of 2

   Czezh

3. In a scalene triangle ABC, AD and BE are angle bisectors. Prove that the acute angle between AB and DE does not exceed $\frac{1}{3}|\angle BAC - \angle ABC|$

   Dusan Djukic

4. The incircle k of a triangle ABC is centered at I and tangent to sides BC, CA, AB at P, Q, R respectively. Lines QR and BC intersect at M. A circle passing through B and C ant tangent to k at point N. The circumcircle of MNP meets the line AP at point L different from P. Prove that I, M, P are collinear

   Djorge Baralic

5. Let $\mathbb{R}^+$ be the set of all positive real numbers. Find all function $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

   $$(1 + yf(x))(1 - yf(x+y)) = 1$$

   for all $x, y \in \mathbb{R}^+$

   Czezh-Polish-Slovak Math Competition

6. There were 64 contestants at a chess tournament. Every pair played a game that ended either one of them or in draw. If a game ended in a draw, then each of the remaining 62 contestants won against at least one of these contestant. There were at least two games ending in a draw at the tournament. Show that we can line up all the contestant so that each of them has won against the one standing right behind him.

   Slovenia TST

7. Let $a, b, c > 0$ and $abc = 1$. Prove that

   $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{3}{a+b+c} \geq \frac{2}{a^2+b^2+c^2}\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)$$

   Komal